Meeting 2 Math 22

Reading Assignment Debrief (8 min)

- Discuss your answer to Task 4 with your group.
- Are there any questions your group wants to address today?

Questions to Address:

- Traces / Contours?
- How to graph a function?

Section 9.1.2 Representing Functions of Two Variables

The most primitive method is to use a table of values:

	... y ...
⋮	⋮
x	... $f(x,y)$ ...
⋮	⋮

A more sophisticated method is to collect all points of the form $(x, y, f(x,y))$.

Definition 9.1.6 The graph of a function $f(x,y)$ is the set of all points $(x, y, f(x,y))$ where (x,y) is in the domain of f. ▨

The table is just a collection of finitely many points on the graph. Usually, we need computers to plot a graph. Try GeoGebra!

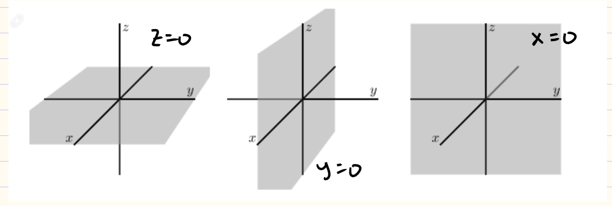
Section 9.1.3 Some Standard Equations in $\mathbb{R}^3$

In $\mathbb{R}^2$, equations of the form $x = c$ or $y = c$ are lines perpendicular to the coordinate axes. We investigate the same equations in $\mathbb{R}^3$.

Activity 9.1.4 (20 min)

- Complete Activity 9.1.4 and discuss w/ your group.
- Class discussion.

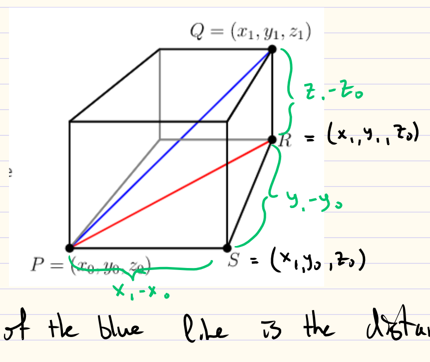
Conclusion: equations like $x = c$ (or $y = c$ or $z = c$) are planes perpendicular to the coordinate axes. When $c = 0$, we get the coordinate planes:



In $\mathbb{R}^2$, a circle is the set of all points equidistant from a fixed point. In $\mathbb{R}^3$, the same definition gives us a sphere. To derive an eq. for a sphere, we need to understand how to compute the distance between two points in $\mathbb{R}^3$.

Activity 9.1.5 (20 min)

- Complete Activity 9.1.5 and discuss w/ your group.
- Class discussion



The length of the blue line is the distance from P to Q. Using Pythagorean Thm twice, we derive the distance formula:

$$|PQ| = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$$

The equation of the sphere centered at (x_0, y_0, z_0) w/ radius $r > 0$ is then

$$r^2 = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2$$

Section 9.1.4 Traces

Definition 9.1.12 A trace of a function $f(x,y)$ in the x direction is a curve defined by the equation

$$z = f(x, c)$$

for some constant $c \in \mathbb{R}$. Similarly, a trace of f in the y direction is a curve of the form

$$z = f(c, y).$$ ▨

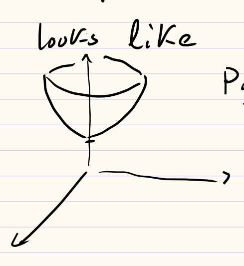
In the next activity, we use traces to determine the graph of a function.

Activity 9.1.6 (20 min)

- Complete parts c, d, and e of Activity 9.1.6 and discuss w/ your group.
- Class discussion.

Traces of $g(x,y) = x^2 + y^2 + 1$ in
- the x-direction: parabolas
- the y-direction: parabolas.

So the graph looks like

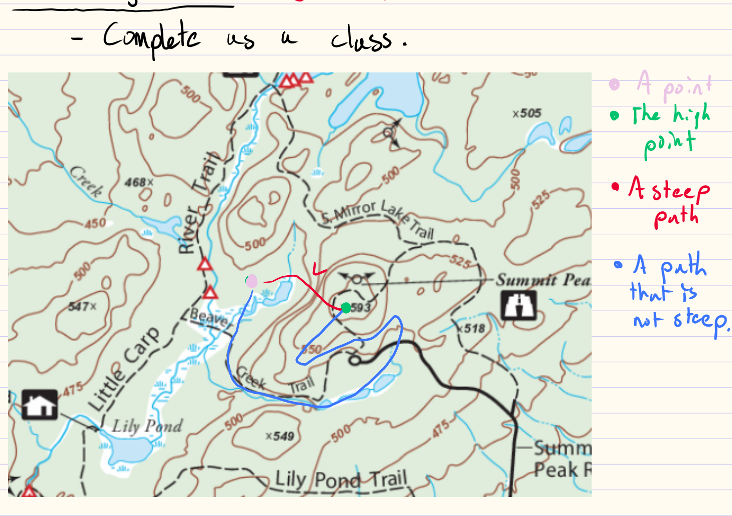


Paraboloid

Section 9.1.5 Contour Maps and Level Curves

Activity 9.1.7 (5 min)

- Complete as a class.



Definition 9.1.15 A level curve or level contour of a function $f(x,y)$ is a curve of the form $K = f(x,y)$ for some constant $K \in \mathbb{R}$. ▨

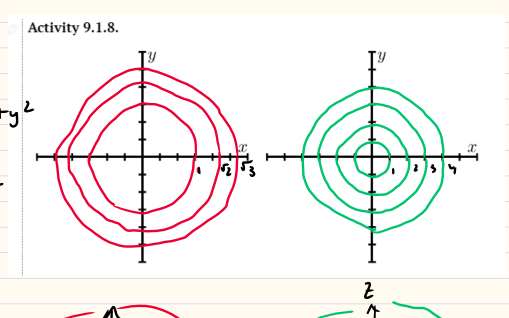
Level curves are the curves obtained by intersecting the graph of f with planes perpendicular to the z axis.

Level curves can also be use to reconstruct the graph of a function.

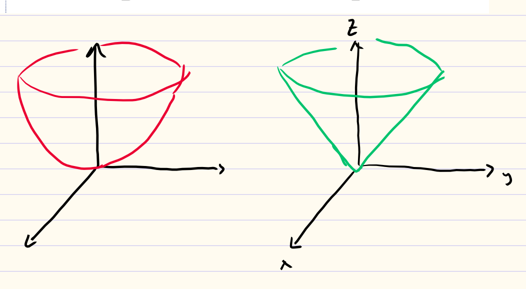
Activity 9.1.8 (20 min)

- Complete Activity 9.1.8 and discuss w/ your group.
- Class discussion.

Level curves for $f(x,y) = x^2 + y^2$ are $K = x^2 + y^2$

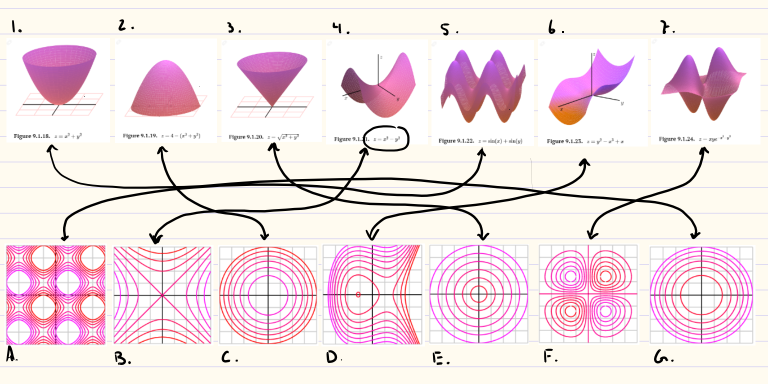


Level curves for $f(x,y) = \sqrt{x^2 + y^2}$



Section 9.1.6 A Gallery of Functions

Match each function with it's level curves:



End of Section 9.1.

Section 9.2 Vectors

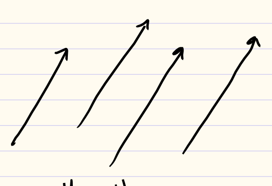
Definition 9.2.1 A vector is a quantity that possesses the attributes of both magnitude and direction.

Examples: • velocity • force • displacement • acceleration

Non-examples: • speed • volume • surface area • length/distance • rate of change of speed

Geometric Representation of Vectors

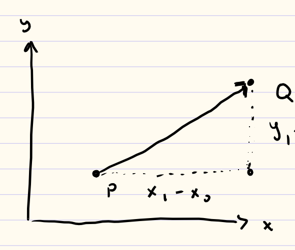
Arrows!



These are all the same vector.

Displacement Vectors Given $P = (x_0, y_0)$ and $Q = (x_1, y_1)$, what is the vector describing the displacement from P to Q? It is given by

$$\overrightarrow{PQ} = \langle x_1 - x_0, y_1 - y_0 \rangle$$



Position Vectors For any point $P = (x, y)$, there is a position vector $\overrightarrow{OP}$ which is the displacement vector from $O = (0,0)$ to P.

$$\overrightarrow{OP} = \langle x - 0, y - 0 \rangle = \langle x, y \rangle$$